MA2331 Homework 1 (to be handed in at 4PM Tuesday October 10 in class)

Questions (all questions have the same weight)

1. Find the Fourier series representation of the sawtooth function f defined by $f(x) = x$ for $0 \leq x < \pi$, $f(x) = 2\pi - x$ for $\pi \leq x < 2\pi$ and $f(x + 2\pi) = f(x)$.

2. Compute
$$\int_{-\pi}^{\pi} dx \ \sin mx \ \sin nx$$
and
$$\int_{-\pi}^{\pi} dx \ \cos mx \ \cos nx$$
for all integer m and n.

3. The periodic function f is defined by
$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$$
and $f(x + 2\pi) = f(x)$.

   (a) Represent $f(x)$ as a Fourier series.

   Remarks: This function is neither odd nor even so both sets of Fourier coefficients are required. However it turns out that all the b_n are zero except for b_1. Can you see why this is the case without computing an integral?

   (b) Derive the remarkable formula
$$\frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \frac{1}{6^2 - 1} + ... = \frac{1}{2}.$$
   It might be useful to consider $f(0)$.

4. Find the half-range Fourier **sine (odd)** expansion of the function $f(x) = 1$ defined on $0 < x < \pi$.