MAU23602 Introduction to Numerical Analysis
5 ECTS credits, Semester 2 module

Contact Hours
11 weeks, 3 lectures per week

Module Coordinator
Professor Kirk Soodhalter
E-mail: ksoodha@maths.tcd.ie

Module Prerequisite
MAU11202 Advanced Calculus

Learning Outcomes
On successful completion of this module, students will be able to:
• explain what numerical analysis is, broadly speaking and understand the floating-point numbering system in which most computers do mathematics; • understand the concepts of conditioning and sensitivity of mathematical problems and how these concepts differ, as well as the formal characterization of a mathematical problem • understand and carry out forward and backward error analysis • understand the theory of polynomial interpolation and implement common interpolation schemes • analyze and implement root finding methods • work with vector/matrix/operator norms and understand their relationship to the singular value decomposition of a matrix • analyze and implement direct methods and stationary iterative methods for the solution of linear equations • analyze and implement numerical integration techniques • analyze and implement numerical methods for solving ODEs

Module Content
Peter Henrici defined numerical analysis as “the theory of constructive methods in mathematical analysis”. In this module, we will learn to analyze and overcome the challenges of constructing often uncomputable analytic quantities when constrained by limitations introduced by the practical world. Often, this is understood to mean the finite precision of a computer. However, this is but a small part of numerical analysis, and the field would exist in the absence of this limitation. The amount of available computer memory limits the size of problems that the computer can solve and also constrains the methods we use to solve these problems. Broadly speaking, there is generally a limitation on the time a computation can take, and we are often constrained by the amount and quality of data we even have about the problem.

More fundamentally, there is a diverse family of continuous problems whose solutions are obtained via analytic limiting procedures, and a core challenge of numerical analysis is to construct accurate approximations to these solutions quickly. Problems of root-finding, quadrature, optimization, and the solution of differential and integral equations, to name a few, all rely on techniques from this field wholly unrelated to the limitations of a computer.

This module serves as an introduction to the challenges of developing and analyzing algorithms for the problems of continuous mathematics. This will be achieved through studying the examples such as polynomial interpolation, root-finding, optimization, and numerical integration as well as methods for solving linear systems of equations.

Recommended Textbooks

There are plenty of books treating the topics of numerical methods, numerical analysis, and numerical linear algebra. The books I am drawing the material for this module from are • Afternotes on Numerical Analysis by G. W. Stewart • Numerical Linear Algebra by Lloyd N. Trefethen and David Bau • Introduction to Numerical Analysis by J. Stoerand R Bulirsch • Numerical mathematics and scientific computation by Björk and Dahlquist

Assessment Detail

Continuous Assessment. A Matlab onboarding assignment will be given in the first week, and assignments which mix analysis and programming will be given roughly fortnightly thereafter.
Final assessment. There will be a take-home final assessment, to be administered online at the end of the semester.
Grading Policy. The homework assignments will count for 50% of the final mark and the final assignment will count for the other 50%.